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TUNNELING OF NEUTRAL PARTICLES THROUGH ONE-DIMENSIONAL FIBONACCI LATTICE

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Abstract. The tunneling of neutral spineless particles through one dimensional Fibonacci lattice has been studied. Using the transfer-matrix formalism, the transmission coefficient of such quasiperiodic structures and also the band structure of the considered system has been calculated.

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1. INTRODUCTION

The scattering of a particle from a potential barrier is one of the most fundamental problems in quantum mechanics. The tunneling exhibited in a scattering process is the most striking quantum phenomenon, which is in conflict with classical intuition. On the other hand, according to Kronig and Penney's seminal article [1], the motion of quantum particle in an infinite periodical structure can explain several interesting properties of real crystals as forbidden energy gaps [2].

A special interest in the motion of neutral spin 1/2 particle through magnetic field is shown in the measurement of the final state polarization in the neutron-spin echo experiments [3,4] and the measurement of the final state of neutron wave function in neutron interferometry [5,6,7]. Also, the change of polarization of neutrino with anomalous magnetic moment during passage through the magnetic field of the Sun is also a possible explanation of the solar neutrino problem [8,9].

The investigation of low dimensional structures has been the subject of study for the last couple of decades because of their possible application in some semiconductor devices, also known as tunneling devices. For that purpose, special class of quasicrystals in the form of Fibonacci superlattice has been studied extensively [10].

2. M-MATRIX FORMALISM

In the beginning, we studied the motion of a spineless particle from the left through a single rectangular potential barrier (Fig. 1) (which is equivalent to the case where the projection of spin is antiparallel to magnetic field), i.e. we consider the time-independent Schrodinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=E\Psi\ , \tag{1}$$

where m is the mass of the particle and the potential energy of magnetic interaction is given by

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 \le x \le a \\ 0, & x > a \end{cases}. \tag{2}$$

In this case $V_0 = \mu_B B$, μB is the magnetic moment and B is the magnetic field.

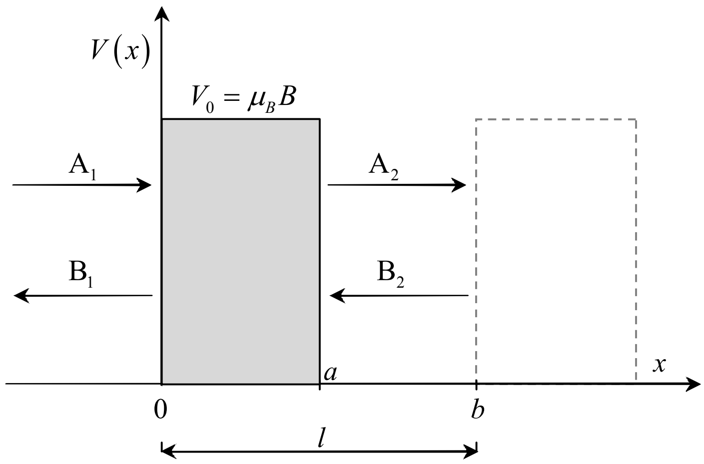

Fig. 1: Magnetic potential barrier of finite width a. In the case of a periodic potential, $l = a + b$ is the period of the potential.

The general solution of the differential equation (1) has a form

$$\Psi(x) = \begin{cases} \Psi_1 = A_1 e^{ik_0 x} + B_1 e^{-ik_0 x} & x < 0 \\ \Psi_2 = C e^{\kappa x} + D e^{-\kappa x} & 0 < x < a \\ \Psi_3 = A_2 e^{ik_0 x} + B_2 e^{-ik_0 x} & x > a \end{cases}, \tag{3}$$

where

$$k_0 = \frac{\sqrt{2mE}}{\hbar}, \text{ and } \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}.$$

The boundary conditions at x=0 and x=a require the relation between amplitudes of the wave functions $\Psi_1(x)$ and $\Psi_3(x)$ to be given by

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = M \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}, \tag{4}$$

where M is especial form of transfer, so-called M-matrix .

The M-matrix formalism is a subject of detailed study in Refs. [11,12], and according to this formalism the transmission coefficient of the wave packet through one-dimensional magnetic lattice is given by

$$T = \frac{1}{\left|(M_N)_{11}\right|^2}, \tag{5}$$

where M_N e is M-matrix for a system of N identical magnetic barriers.

The band structure of the energetic specter of the neutral spineless particles according to *M*-matrix formalism is calculated by the expression $\cos\gamma = Tr\,P/2$, where γ is a real parameter and the *P* matrix is given by the expression

$$P = \begin{pmatrix} M_{22}e^{ik_0 l} & M_{21}e^{ik_0 l} \\ M_{12}e^{-ik_0 l} & M_{11}e^{-ik_0 l} \end{pmatrix}. \quad (7)$$

3. SYSTEMS OF QUASIPERIODIC ONE-DIMENSIONAL MAGNETIC LATTICE

3.1. Fibonacci lattice

Many theoretical studies [13] have special interest (for) in the quasiperiodic 1D lattices and that interest stems partly from the fact that the Bloch theorem cannot be applied to them, and also because the quasiperiodic lattices are an intermediate case between periodic and disordered 1D solids. Quasiperiodicity leads to a special kind of energetic spectra, known as Cantor sets, characterized by localized and extended eigenstates, but also by singular continuous components with chaotic extended states [13].

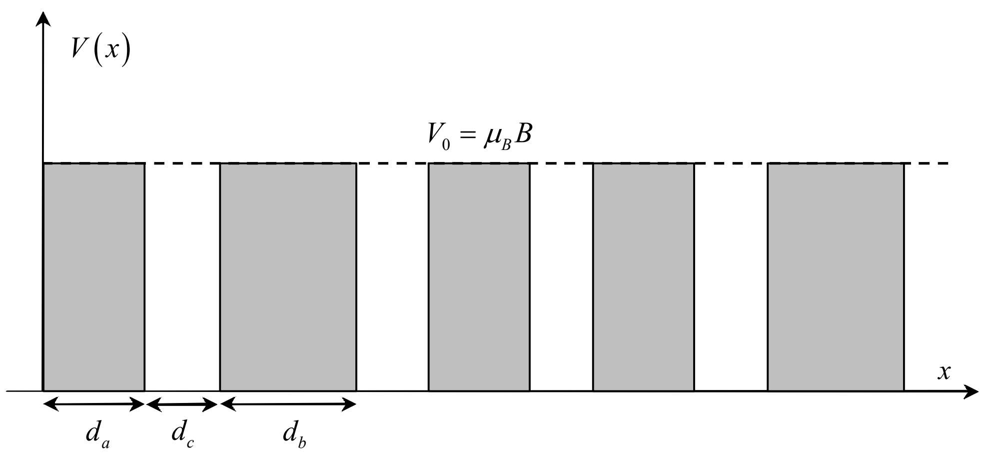

Fig. 2: 1D magnetic quasiperiodic lattice generated by two segments in the form of a rectangular barriers. The width of the barriers is d_a and d_b and separation between them is d_c.

Following Levine and Steinhardt's ideas [10], in this paper we have investigated a special class of 1D magnetic quasi lattices built by two building blocks with different thickness d_a and d_b. The energetic height of both barriers is $V_0 = \mu_B B$, as is shown in figure 2. Fibonacci lattices are generated by the expression $z_{i+1} = z_i + r_i$, where z_i is a coordination number, and $\{r_i\}$ is Fibonacci sequence of intervals $\{d_a d_b d_a \cdots\}$. The proportion of the building blocks' thickness, given by $d_a/d_b = \tau \approx 1,618$ is known as golden mean [14].

3.2. Transmission coefficient and band structure

In this study, the calculation of the transmission coefficient and the band structure of 1D magnetic quasi lattice is also done by the M-matrix formalism. For this purpose, we have written *Mathematica* code. The separations between the barriers are $d_c = d_b = 1$, and the energy of the particles is reduced by $\mu_B B$. This leads to $V_0 = 1$.

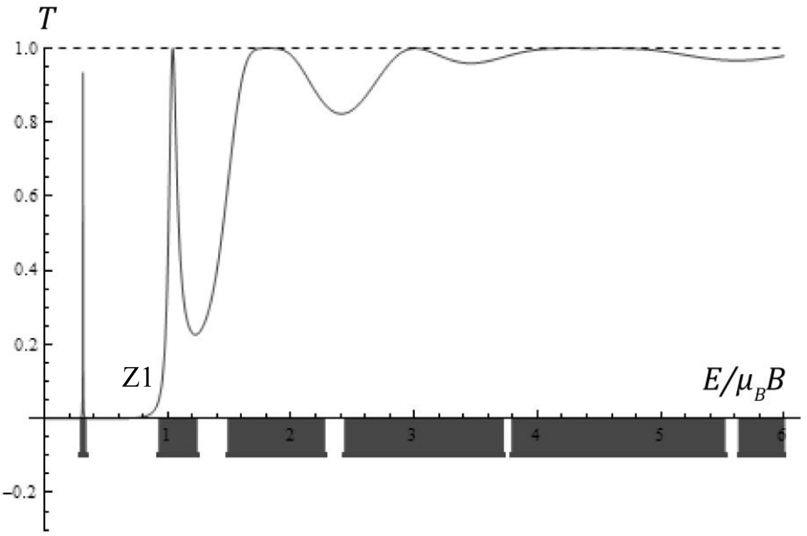

Fig. 3a.

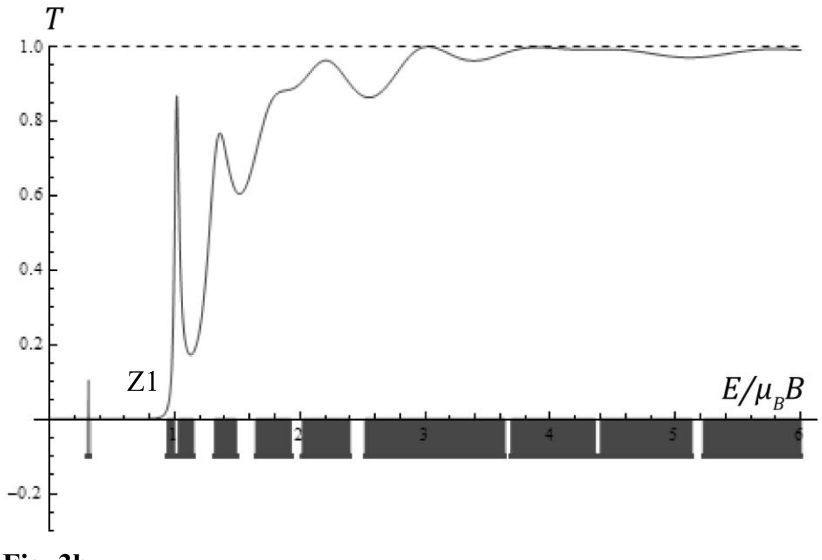

Fig. 3b.

Figures 3 and 4 show the coefficients of transmission and the band structures of the Fibonacci magnetic lattices with two and five barriers, successively. In figures 3a and 4a the thickness of the barriers is the same, $d_a = d_b$, whereas in figures 3b and 4b we have Fibonacci sequences with $d_a / d_b = 1{,}618$.

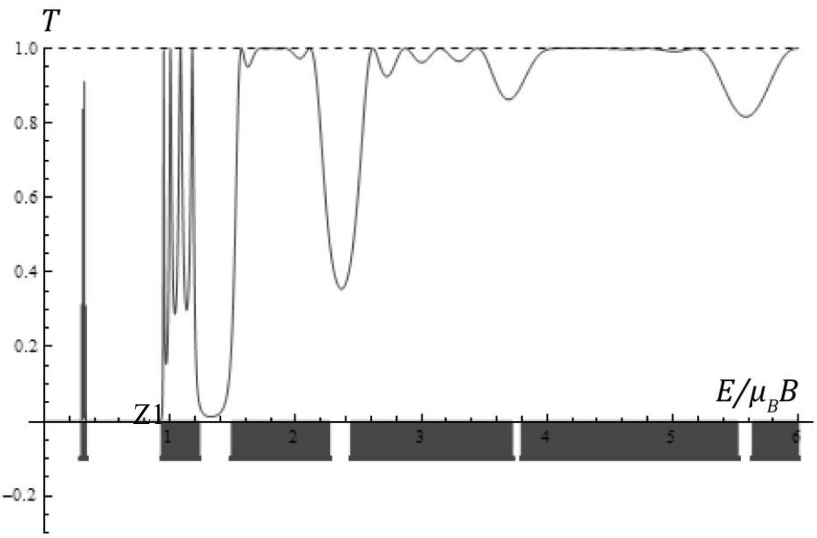

Fig. 4a.

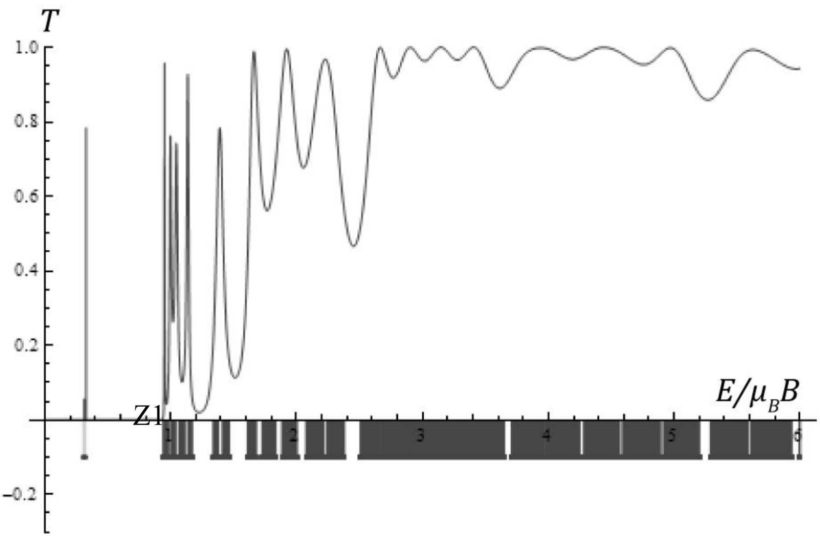

Fig. 4b.

4. CONCLUSIONS

The tunneling of neutral spineless particles through some magnetic configurations has been investigated. It is shown that the transfer-matrix method is powerful for the investigation of the periodical and the aperiodical structures.

The transmission coefficients analysis, and the analysis of the band structures of magnetic lattices, as presented in figures 3b and 4b, clearly show us that localized eigenstates appear as a result of the resonant tunneling. In both cases, we have one resonant level approximately $E_0 \approx 0{,}3$, whereas the zones Z1 are divided into subzones. The coefficients of transmission have n-1 peaks, where n is the number of barriers (two or five), which are typical nonunity transmission resonances.

This behavior of the coefficient of transmission is expected, because it is a well-known characteristic of one-dimensional structures with asymmetrical thickness of the barriers.

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ТУНЕЛИРАЊЕ НА НЕУТРАЛНИ ЧЕСТИЦИ НИЗ БАРИЕРИ ОД МАГНЕТНИ ПОЛИЊА

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Апстракт. Проучено е тунелирање на неутрална честица низ конфигурации на магнетини бариери во форма на еднодимензионални решетки на Фибоначи. Користејќи го формализмот на трансфер матрици, пресметани се коефициентот на трансмисија како и нивната зонска структура. Од добиените резултати може да се заклучи дека коефицентот на трансмисија има резонантни пикови за кои $T = 1$ но и резонантни пикови за кои $T < 1$. Зонската струкатура е сложена и има карактеристики на апериодични кристални решетки-квазикристали.