Tunneling of neutral particles through barriers of magnetic fields

B. Veljanoski and N. Stojanov

Institute of Physics, Faculty of Natural Sciences and Mathematics,
Sts. Cyril and Methodius University, P.O.Box 162, 1001 Skopje, Macedonia

PACS. 03.65.Ge – Solutions of wave equations: bound states.
PACS. 03.65.Nk – Scattering theory.
PACS. 03.65.Xp – Tunneling, traversal time, quantum Zeno dynamics.

Abstract. – The tunneling of neutral spinless particles through some configurations of magnetic potential barriers is studied. Using the transfer-matrix formalism, the transmission coefficient of such periodical and aperiodical structures and also the band structure of the considered systems are calculated.

Introduction. – Scattering of a particle from a potential barrier is one of the most fundamental problems in quantum mechanics. The tunneling exhibited in a scattering process is the most striking quantum phenomena in conflict with classical intuition.

On the other hand, according the seminal article of Kronig and Penney [1], the motion of quantum particle in an infinite periodical structure can explain several interesting properties of real crystals as forbidden energy gaps. The investigation of low dimensional structures has been the subject of study for the last couple of decades because of their possible application in semiconductor microstructures [2, 3].

The special interest is a motion of neutral spin 1/2 particle through magnetic field in the connection with the measurement of the final state polarization in the neutron-spin echo experiments [4, 5] and the measurement of the final state of neutron wave function in neutron interferometry [6–8]. Also the change of polarization of neutrino with anomalous magnetic moment during passage through magnetic field of Sun is one possible explanation of solar neutrino problem [9, 10].

The rectangular magnetic barrier. – In the beginning we study the motion of a spinless particle from the left through a single rectangular potential barrier (fig. 1) (which is equivalent to case that the projection of spin is antiparalel to magnetic field), i.e. we consider the time-independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\Psi=E\Psi, \tag{1}$$

with

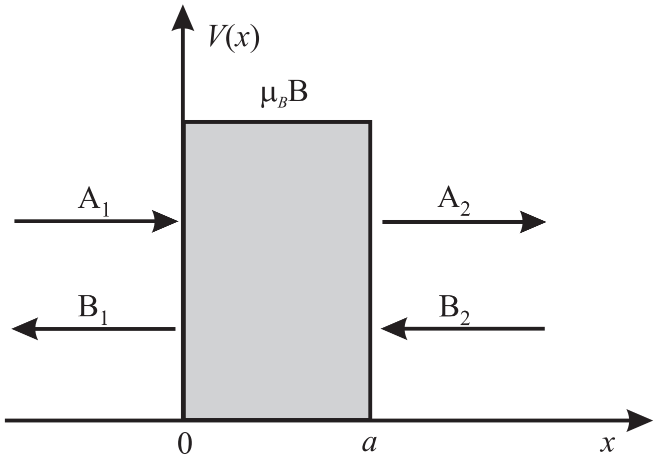

Fig. 1 – Single magnetic potential barrier of finite width a.

$$V(x) = \begin{cases} 0, & x < 0 \\ \mu_B B, & 0 \le x \le a \\ 0, & x > a \end{cases} \tag{2}$$

where m is the mass of the particle, μ_B is the magnetic moment and B is the magnetic field.

In the regions where $V(x)$ is zero the general solution $\Psi(x)$ of the differential equation (1) has a plane wave form

$$\begin{aligned} \Psi_1 &= A_1 e^{ik_0 x} + B_1 e^{-ik_0 x}, \quad x < 0, \\ \Psi_2 &= A_2 e^{ik_0 x} + B_1 e^{-ik_0 x}, \quad x > a, \end{aligned} \tag{3}$$

where the relation between particle energy E and wave vector k_0 is

$$k_0 = \frac{\sqrt{2mE}}{\hbar}.$$

The boundary conditions at $x = 0$ and $x = a$ require the relation between amplitudes of the wave functions $\Psi_1(x)$ and $\Psi_2(x)$

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \tag{4}$$

where M is especial form of transfer matrix [11]. For practical purpose it is convenient to represent a M-matrix via the product of two types of matrices: discontinuity matrix and propagation matrix [12], i.e.

$$M = d_1^0 p_{1,-a} d_0^1 p_{0,a} \tag{5}$$

where

$$d_0^1 = \frac{1}{2}\begin{pmatrix} 1+k_0/k_1 & 1-k_0/k_1 \\ 1-k_0/k_1 & 1+k_0/k_1 \end{pmatrix}, \quad d_1^0 = \frac{1}{2}\begin{pmatrix} 1+k_1/k_0 & 1-k_1/k_0 \\ 1-k_1/k_0 & 1+k_1/k_0 \end{pmatrix} \tag{6}$$

with

$$k_1 = \frac{\sqrt{2m(E-\mu_B B)}}{\hbar}$$

are the discontinuity matrices and

$$p_{0,a} = \begin{pmatrix} e^{ik_0a} & 0 \\ 0 & e^{-ik_0a} \end{pmatrix}, \quad p_{1,-a} = \begin{pmatrix} e^{-ik_1a} & 0 \\ 0 & e^{ik_1a} \end{pmatrix} \tag{7}$$

are the propagation matrices.

When we perform the matrix multiplication in the formulae (5-7) we obtain finally

$$M = \begin{pmatrix} M_{11}^1 & M_{12}^1 \\ M_{21}^1 & M_{22}^1 \end{pmatrix}, \tag{8}$$

where the following notation has been introduced, $(j = 1, 2)$

$$M_{11}^1 = e^{i\mu_1}\cosh\lambda_1, \qquad M_{12}^1 = i\sinh\lambda_1,$$

$$M_{21}^1 = -i\sinh\lambda_1, \qquad M_{22}^1 = e^{-i\mu_1}\cosh\lambda_1,$$

$$\mu_1 = k_0a - \arctan(\frac{\eta_1}{2}\tan k_1a), \qquad \lambda_1 = -\sinh^{-1}(\frac{\varepsilon_1}{2}\sin k_1a), \tag{9}$$

$$\varepsilon_1 = \frac{k_1}{k_0} - \frac{k_0}{k_1}, \qquad \eta_1 = \frac{k_1}{k_0} + \frac{k_0}{k_1}.$$

Then the transmission coefficient is

$$T = \frac{1}{|M_{11}^1|^2}\,. \tag{10}$$

Transmission coefficient. – Let us study the scattering states of a spinless particles which incidents from the left and transmits through N rectangular barriers of width a separated by the gap c (the period has a length $l = a + c$) (fig. 2).

The elementary analysis shows that the transfer matrix of periodical lattice in fig. 2 can be represent as a product of discontinuity matrices (6) and propagation matrices (7), i.e.

$$M_N = d_1^0 p_{1,-a} d_0^1 p_{0,-c} d_1^0 p_{1,-a} d_0^1 p_{0,-c} \cdots d_1^0 p_{1,-a} d_0^1 p_{0,-c} p_{0,N(a+c)} \tag{11}$$

or

$$M_N = t^N p_{0,N(a+c)} \tag{12}$$

where we introduce a new transfer matrix t by

$$t = d_1^0 p_{1,-a} d_0^1 p_{0,-c}. \tag{13}$$

In fig. 3 is shown a graphical representation of M_N-matrix. It is evident from (5), (7) and (13) that connection between M-matrix and t-matrix is

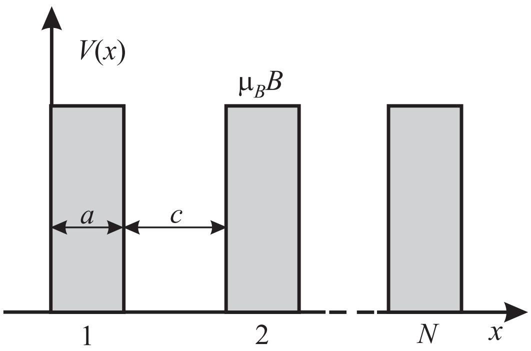

Fig. 2 – System of rectangular magnetic barriers.

$$t = \begin{pmatrix} M^1_{11} e^{-ik_0 l} & M^1_{12} e^{ik_0 l} \\ M^1_{21} e^{-ik_0 l} & M^1_{22} e^{ik_0 l} \end{pmatrix}. \tag{14}$$

Then M_N-matrix of the periodical system of barriers can be expressed via the elements of N-th degree of t-matrix

$$M_N = \begin{pmatrix} (t^N)_{11} e^{ik_0 Nl} & (t^N)_{12} e^{-ik_0 Nl} \\ (t^N)_{21} e^{ik_0 Nl} & (t^N)_{22} e^{-ik_0 Nl} \end{pmatrix}. \tag{15}$$

The transmission coefficient is

$$T = \frac{1}{|(M_N)_{11}|^2} = \frac{1}{|(t^N)_{11}|^2}\,. \tag{16}$$

Of course, the N-th degree of t-matrix (14) can be performed analytically [13], but for our aims in this paper it is not necessary. Also, our result can be extended to spin 1/2 particle [13] without any difficulty.

The analytical form of t-matrix directly determine the band structure of an infinite lattice. Namely, the eigenvalues of t are roots of the eigenvalue equation

$$x^2 - x\mathrm{Tr}t + \det t = 0\,. \tag{17}$$

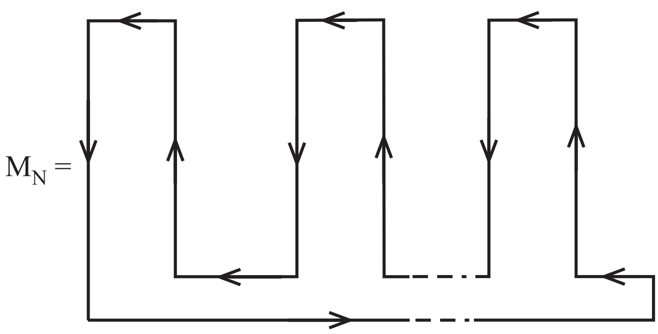

Fig. 3 – Graphical representation of M_N-matrix.

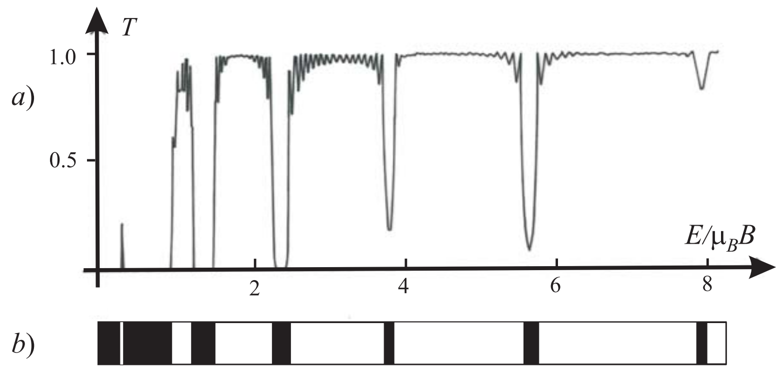

Fig. 4 – a) The transmission coefficient of the system of rectangular magnetic potential barriers as a function of reduced energy $E/(\mu_B B)$ for fixed values: $N = 20$, $a = b = 1$, and $\rho = a(2m\mu_B B)^{1/2}/\hbar = \pi/0.9$. b) The band structure of an infinite chain of rectangular potential barriers for fixed values: $a = b = 1$, $\rho = \pi/0.9$.

Because of [11] $\det t = 1$, the requirement that eigenvalues are purely imaginary (in this case the wave function remains finite) leads to the equation of band structure

$$\frac{1}{2}|\mathrm{Tr}t| \leq 1 \tag{18}$$

In fig. 4 is shown the transmission coefficient (16) of great number of rectangular barriers and the band structure (17) of an infinite lattice. It is evident that the position of forbidden zones are the same as positions of minimal transmission coefficient. So we can propose that "band" structure of aperiodical lattice is possible to reconstruct by the form of transmission coefficient of large number of barriers.

Magnetic sandwich structure. – The above formalism enable us to investigate the more complex structures. Below we employ the one-dimensional model of two lattice of rectangular magnetic barriers (fig. 5) having a contact. The left hand lattice contents N_1 barriers of width a separated by the gap c. The right hand lattice contents N_2 barriers of width b separated by the gap d.

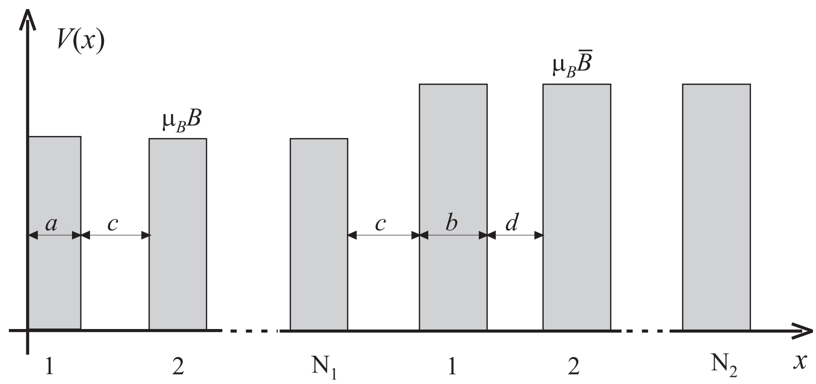

Fig. 5 – Magnetic sandwich structure.

The approach of the above formalism leads to the $M_{N_1+N_2}$ matrix

$$M_{N_1+N_2} = t^{N_1}\bar{t}^{N_2} p_{0,N_1 l+N_2\bar{l}} \tag{19}$$

where $l = a + c \quad \bar{l} = b + d$, and

$$\begin{aligned} t &= d_1^0 p_{1,-a} d_0^1 p_{0,-c}\,, \\ \bar{t} &= d_2^0 p_{2,-b} d_0^2 p_{0,-d}\,. \end{aligned} \tag{20}$$

In the formulae (19) and (20) we introduce the discontinuity matrix and the propagation matrix of right hand side lattice

$$d_0^2 = \frac{1}{2}\begin{pmatrix} 1+k_0/\bar{k}_2 & 1-k_0/\bar{k}_2 \\ 1-k_0/\bar{k}_2 & 1+k_0/\bar{k}_2 \end{pmatrix}, \quad p_2^b = \begin{pmatrix} e^{i\bar{k}_2 b} & 0 \\ 0 & e^{-i\bar{k}_2 b} \end{pmatrix}, \tag{21}$$

where $\bar{k}_2 = \sqrt{2m(E - \mu_B \bar{B})}/\hbar$.

Then the transmission coefficient can be calculated from the formula

$$T = \frac{1}{|(t^{N_1}\bar{t}^{N_2})_{11}|^2}\,. \tag{22}$$

In fig. 6a is shown the plot of transmission coefficient for the considered sandwich system. From this graph we can conclude that the energy "band" structure for this system (for which there is not the analytical expression because the Bloch condition is not satisfied) is as in fig. 6b. In figs. 6c and 6d are shown the forbidden energy gaps of infinity left hand lattice and right hand lattice.

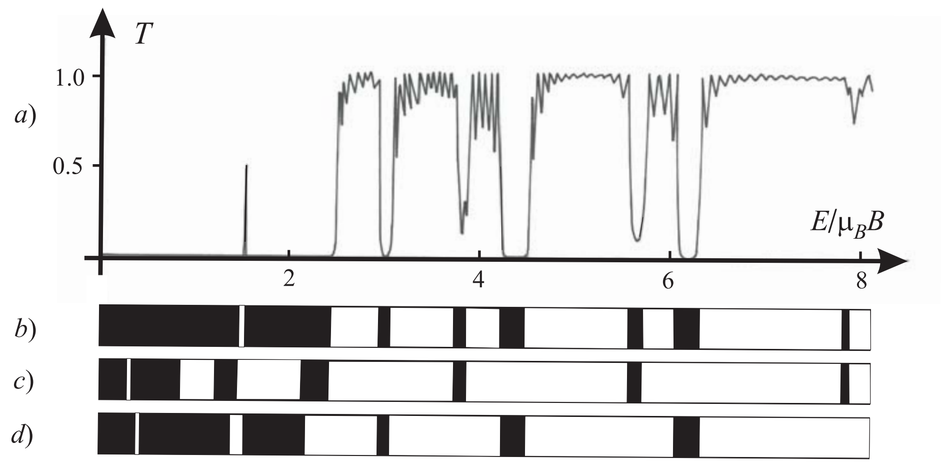

Fig. 6 – a) The transmission coefficient of the sandwich structure as a function of reduced energy $E/(\mu_B B)$ for fixed values: $N_1 = N_2 = 20$, $a = b = c = d = 1$, $\rho = a(2m\mu_B B)^{1/2}/\hbar = \pi/0.9$ and $\bar{B} = 2B$. b) The energy band structure for the magnetic sandwich structure. c) The energy band structure for the left hand lattice. d) The energy band structure for the right hand lattice.

From these graphics is evident that an addition rule is valid: the zones of forbidden energy of such sandwich structures are a summa of zones of forbidden energies of each lattice.

Conclusion. – The tunneling of neutral spinless through some magnetic configurations has been investigated. It is shown that the transfer-matrix method is powerful in consideration of periodical and also aperiodical structures. The main result is that forbidden energy gap of contact sandwich structure is an addition of energy gaps of particular sublattices.

REFERENCES

[1] KRONIG R. and PENEY W.G., *Proc. R. Soc. Lond. Ser. A*, **130** (1930) 499.
[2] PANDEY L.N. and GEORGE T.F., *Appl. Phys. Lett.*, **61** (1992) 1081.
[3] STOKMAN M.I. *et. al.*, *Phys. Rev. B*, **45** (1992) 8550.
[4] MEZEI F., *Z. Phys.*, **255** (1972) 146.
[5] REKVELDT T.TH., *Z. Phys.*, **259** (1973) 391.
[6] RAUCH H. *et. al.*, *Phys. Lett. A*, **55** (1975) 425.
[7] WERNER S.A. *et. al.*, *Phys. Rev. Lett.*, **35** (1975) 1053.
[8] KLEIN A.G. and OPAT G.I., *Phys. Rev. Lett.*, **37** (1976) 238.
[9] VOLOSHIN M.B. *et. al.*, *Sov. J. Nucl. Phys.*, **44** (1986) 440.
[10] VOLOSHIN M.B. and VYSOTSKII M.I., *Sov. J. Nucl. Phys.*, **44** (1986) 544.
[11] MERZBACHER E., *Quantum Mechanics* (Wiley, New York) 1970
[12] WALKER J.S. and GATHRIGHT J., *Am. J. Phys.*, **62** (1994) 408.
[13] MIJATOVIC M. *et. al.*, *Z. Phys. A*, **345** (1993) 65.